

... to discover this process.

9.11 CONSTITUTION OF NUCLEUS AND NUCLEAR SHELL MODEL

The atomic nucleus is composed from two elementary particles protons and neutrons that interacting through the strong nucleon-nucleon forces. The atomic nucleus is a typical example of a complex system that exhibits on one side very simple modes of motion such as vibrations, rotations etc. next to very complex excitations.

A basic approach to study the atomic nucleus is to make use of quantum mechanical Hartree-Fock technique that introduces a field of average potential in which the nucleons can move independent particles like that of electrons move in average coulomb field of atom. This picture allows us to bring a structure of nuclear shell model in order to describe and explore a multitude of nuclear properties.

Most nucleus are considered to be spherical which can be judged from the electric quadrupole moment of the nucleus. The nucleons i.e. protons and neutrons have their nuclear spin and nuclear magnetic moment. If the quadrupole moment is zero the nucleus is spherically symmetrical. On the other hand positive and negative values of the moment indicates non-spherical nature such as **prolate** or an

Shape of the nucleus

* All elementary particles such as electrons, protons, neutrons, neutrino etc. have parity factor which denotes the sign of the wave function as related to symmetry properties.

oblate (Fig. 9.9) type ellipsoidal shape. It is to be noted that even number of protons and even number of neutrons form the nucleus having zero spin and nuclear magnetic moment. Therefore paired number of nucleons have greater nuclear stability compared to that of odd number of protons and even number of neutrons or vice-versa which have non zero magnetic moment.

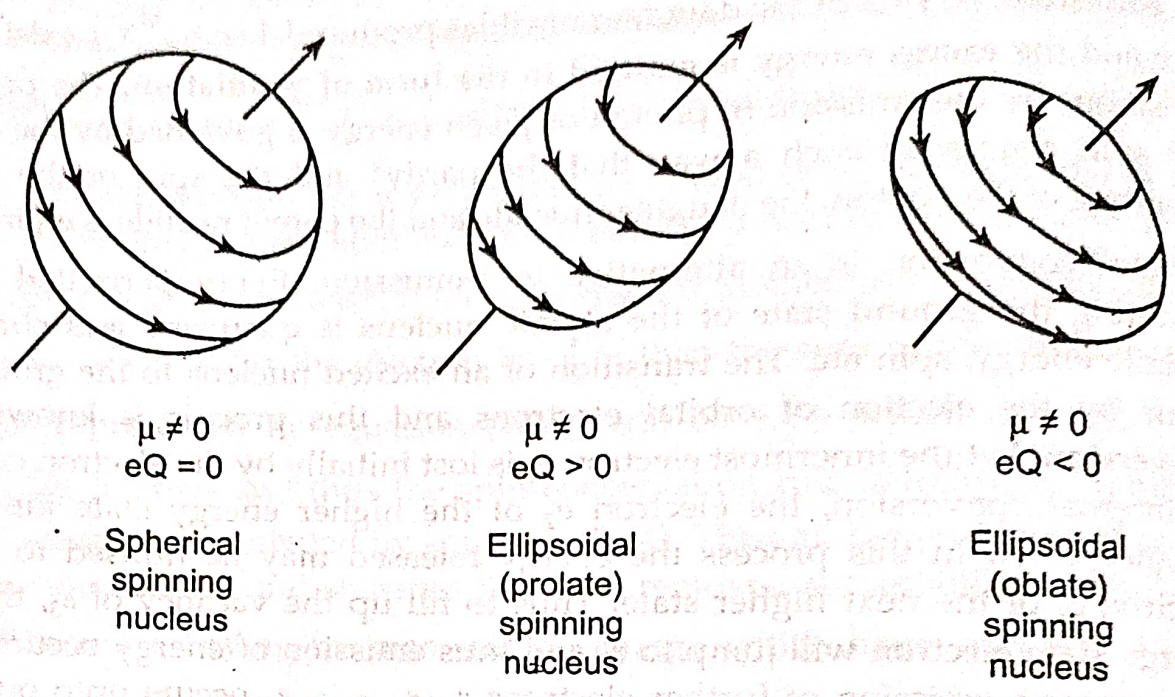


Figure 9.9 : Shape of the nucleus. μ = magnetic moment, eQ = electric quadrupole moment.

In order to explain certain fundamental questions about the composition of the nucleus such as (a) what type of force holds between the nucleons (b) how α , β and γ -rays are emitted from the nucleus having composition with protons and neutrons (c) why some nuclides are stable and some are unstable. In order to explore several questions of nuclear stability several scientists proposes different **nuclear shell model** to explain different properties of the nucleus to fit with the experimental data. The proposed models are—

- (1) Early closed shell model developed in 1932-36.
- (2) Niels Bohr and Frenkel **liquid drop model** (1936-48). This model suggests that a nucleus is a homogeneous entity of nucleons with strong interaction among all the neighbour nucleons similar to that of a liquid drop.
- (3) **Independent particle model** of M.G. Mayer (1948-50).
- (4) **Statistical Fermi gas model** which treats the nucleons statistically as a whole and the concept of **nuclear potential** developed.
- (5) **Collective model** developed by A. Bohr and B.R. Mottelson (1951-53). This model treats the movement of the nucleus as a whole as also the movement of the unpaired nucleons.

We shall discuss here one or two models for the sake of clarity.

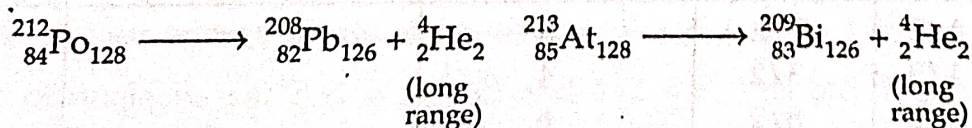
9.11.1 Closed shell model and magic numbers

This nuclear shell model deals with the nucleus in the ground state. Here nucleons are distributed in some discrete quantum mechanical energy levels like that of electrons. The capacity of each level is related to certain numbers, viz. 2, 10, 18, 36, 54 and 86 electron numbers like that of electronic energy levels. These numbers form the closed shell electronic configuration (He, Ne, Ar, Kr, Xe and Rn) of each period and thus this model is called closed shell model.

It is well known that periodic properties are intimately related to those numbers which forms the basis of periodic classification of elements. In a similar fashion the nuclear properties, viz. stability, binding energy* etc. are also related to vary in each period in which the nuclear periods end with the number of protons or neutrons denoted by 2, 8, 20, 50, 82 or 126. The numbers 114, 164 and 184 may also be included in the list. These numbers are commonly called **magic numbers** and these nuclides have exceptional stability. The importance of these numbers are:

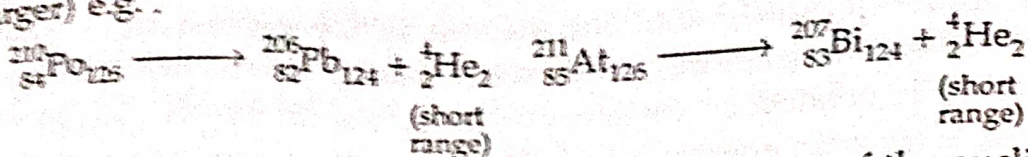
Magic number and its importance

- Isotopic species having magic number of protons and neutrons will show **exceptional stability** i.e. ${}^4_2\text{He}_2$, ${}^{40}_{20}\text{Ca}_{20}$, ${}^{16}_8\text{O}_8$, ${}^{88}_{38}\text{Sr}_{50}$, ${}^{138}_{56}\text{Ba}_{82}$ etc.
- It has been stated that **heaviest non radioactive nuclide in nature** is ${}^{209}_{83}\text{Bi}$ having 126 neutrons. It is also evident that all the end products of three naturally occurring disintegration series is the isotopes of lead i.e. ${}^{206}_{82}\text{Pb}$, ${}^{207}_{82}\text{Pb}$ and ${}^{208}_{82}\text{Pb}$. Possibly after ${}^{83}\text{Bi}$ why ${}_{82}\text{Pb}$ having magic number protons (82) is the end product of all disintegration series.
- It is to be noted that **mean binding energy per nucleon** is maximum for the nuclides with both magic number neutrons and protons are ${}^4_2\text{He}_2$, ${}^{16}_8\text{O}_8$, ${}^{40}_{20}\text{Ca}_{20}$ and ${}^{208}_{82}\text{Pb}_{126}$.
- Relative abundance of elements in earth's crust can also be denoted by the magic numbers e.g. ${}^{16}_8\text{O}_8$, ${}^{88}_{38}\text{Sr}_{50}$, ${}^{89}_{39}\text{Y}_{50}$, ${}^{90}_{40}\text{Zr}_{50}$, ${}^{208}_{82}\text{Pb}_{126}$ etc. Magic number of protons and neutrons composing nuclides are considered as of terrestrial or cosmic origin.
- Alpha emission** means the decrease of atomic number by two units i.e. mechanistically emission of helium nucleus having two protons and two neutrons. Thus the nuclide having 128 neutrons would favour alpha decay and thus converted to the magic number nuclides e.g.



These alpha particles are of high energy and short-lived ($t_{1/2}$ is smaller) in accordance with the Geiger-Nuttall rule (Sec. 9.10.2). On the other hand the nucleus having magic

number of neutrons i.e. 126 will emit alpha particles of lower energy and long-lived ($t_{1/2}$ larger) e.g. .



(f) Beta emission can also be associated with the conversion of the nuclide having magic number of neutrons or protons e.g.



These beta particles are of high energy and short lived.

(g) Nuclear cross section* for nuclear transmutation reactions* with neutrons as projectile has importance with magic numbers. The cross section is lowered for nuclides having 20, 50, 82 and 126 number neutrons compared to that of neighbouring nuclide having one neutron less than that of magic number. A high value of nuclear cross section indicates greater efficiency of nuclear reactions.

9.11.1.1 Explanation of magic numbers in terms of nucleon pairing like that of electrons

We have stated that nucleons i.e. protons and neutrons will assume some quantised energy states like that of electrons. Like electrons periodicity of nuclear properties may exist at magic number of neutrons and protons where shell closure occurs. Electrons tend to pair to form a stable chemical bond so also nucleons do so to form the nuclear stability. Like electrons the

Table 9.7 : Filling of energy levels by nucleons

State	l	s	$j = l+s$ $j = l-s$	Total number of nucleons ($2j+1$)	Total number of nucleons including the former levels	Alternative state like that of electronic state i.e. $2s+1L_J$
1s	0	1/2	1/2	2	2	$1s_{1/2}$
1p	1	1/2	3/2 1/2	4 2	8	$2p_{3/2}$ $2p_{1/2}$
1d	2	1/2	5/2	6		$3d_{5/2}$
2s	0	1/2	1/2	2		$2s_{1/2}$
1d	2	1/2	3/2	4	20	$3d_{3/2}$
1f	3	1/2	7/2	8		$4f_{7/2}$
2p	1	1/2	3/2	4		$3p_{3/2}$
1f	3	1/2	5/2	6		$4f_{5/2}$
2p	1	1/2	1/2	2		$3p_{1/2}$
1g	4	1/2	9/2	10	50	$3g_{9/2}$

* will be discussed later



- (a) The nucleus is considered as a homogeneous entity of the neutrons and protons like that of a liquid drop of pure liquid or solution where molecules or atoms exist.
- (b) Each liquid has a certain latent heat of vaporisation which is similar to that of binding energy of the nucleons to form the nucleus.
- (c) Each nucleon in a nucleus behaves in the same way as that of the molecules in a liquid drop. Each nucleon interacts strongly with the neighbours and the interaction energy is proportional to mass number (A).
- (d) Both the liquid drop and the nucleus is incompressible. In a homogeneous liquid drop and nucleus, the density, charge and other properties will be the same throughout the liquid drop and the nucleus except at the surface.
- (e) In a liquid drop and in nucleus the molecules or atoms and the nucleons interact themselves respectively. The interacting forces in the nucleus are $p-p$, $p-n$ and $n-n$. These nuclear forces are short range force and of saturation type operated within the nuclear dimension of the order of $\sim 10^{-13}$ cm. If all the nucleons interact in the same way the binding energy of the nucleus per nucleon will be very high. It has been suggested that for an effective formation of a liquid drop or a nucleus the molecules and the nucleons interact in the same way. In a stable nucleus the attractive force predominates over the repulsive force and all the exchange attractive forces between $p-p$, $p-n$ and $n-n$ are of same type and independent of charge.
- (f) Like that of a liquid drop the spherical nucleus also exhibits surface tension force. The surface tension force is proportional to the surface area of the nucleus.
- (g) Similar to a liquid drop a nucleus may capture high energy particle or projectile to form an excited compound nucleus. The excess energy of the projectile is equally distributed among the nucleons of the nucleus or the liquid drop.
- (h) The compound nucleus or the excited liquid drop may be deexcited by the following processes :
- (1) By emission of radiation similar to cooling or by removing heat from the excited liquid drop.
 - (2) Emission of one or more particles from the compound nucleus like that of evaporation of some molecules from the liquid drop.
 - (3) By nuclear fission into two fragments like that of the excited liquid drop breaks into two parts.
- (i) The common feature of the union of the two smaller liquid droplets to form a bigger liquid drop and the reverse that is the breaking of a larger liquid drop into two smaller units similarly may be followed in case of two lighter nuclei undergoing nuclear fusion while one heavier nuclei disintegrated by high energy projectile into two or more smaller nuclei in the case of nuclear fission.

The important aspect of this shell model is that it provides satisfactory explanation of nuclear fission by the formation of excited compound nucleus (other shell model deals with ground state nuclei). It also provide some method of calculating binding energy of the

nucleons. The values obtained are in good agreement with that of experimental data and this provide experimental evidence in favour of liquid drop model.

*Merits and demerits
of liquid drop model*

The limitation of this model is that it does not provide explanation of extra stability of **magic number** nuclides with closed shell effects. This also inadequate to explain nuclear forces for the independent motion of nucleons, as also their spin, parity and magnetic property.

11.3 The nuclear radius

9.11.4 Nuclear forces and meson

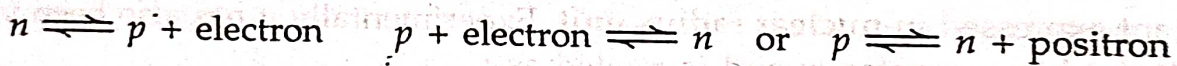
As stated previously (Sec. 9.11 and 9.11.2) that nuclear forces are mainly proton-proton ($p-p$), proton-neutron ($p-n$) and neutron-neutron ($n-n$) forces operated within small nuclear radius ($\sim 10^{-15}\text{m}$). Characteristic of these forces are :

(a) **Different from gravitational or electrostatic forces** : It is evident that if it is a mere

electrostatic force then the $p-p$ force will be repulsive. But comparing the binding energies of various nuclides it is established that all the forces i.e. $p-p$, $p-n$ and $n-n$ forces are of equal magnitude and these forces are much stronger than electric and gravitational attractive forces.

- (b) **Short range type** : These forces are not of long range gravitational or electrostatic type but of short range type as stated before. Outside the nuclear dimension these forces have no real existence.
- (c) **Charge independent** : All the nuclear forces contribute equally to the nuclear stability irrespective of the $p-p$ repulsive force. This is verified on the fact that binding energy per nucleon is more or less constant as also nuclear density (ρ). Moreover in case of mirror nuclei* the replacement of a $p-p$ force by a $n-n$ force does not change the nuclear force. Thus we conclude nuclear forces are charge independent.
- (d) **Saturation type** : As like valency saturation nuclear forces are of saturation type. This is indicated by the constancy of nuclear binding energy per nucleon (~ 8.5 Mev).

Thus it is evident that nature of the nuclear forces are very complex. The next question arises how neutron and proton bind to form stable nucleus. Moreover there is a formidable force of attraction between neutron and proton and repulsion between proton and proton. As stated above nuclear forces are of saturation type like that of the chemical forces. So from quantum mechanical calculation Heisenberg proposed that strong force operated between the nucleons is due to the exchange phenomena operated between the nucleons because exchange operation is well established in quantum mechanics. Since the forces are of short range type so the exchanging particle should have finite mass and thereby Heisenberg proposed that electron is the exchanging particle between the nucleons. Thus a neutron exchanging to proton by emission of electron while a proton converted to neutron by the capture of electron or emission of positron i.e.



Rejection of exchanging particle as electron

However this proposition has serious objection. Calculations reveal that if electron is taken as the exchanging material then the nature of the nuclear forces will be of more longer range as also the forces

will be weaker.

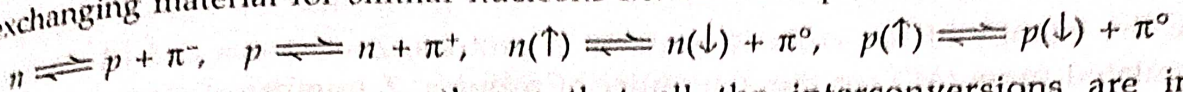
In order to obviate this difficulty Hideki Yukawa (1935), a Japanese physicist proposed a new exchanging particle which is responsible for the strong short range forces operated between the nucleons and named the particle as meson**. He proposed three types of $\pi(\pi)$ mesons viz.

* will be discussed later.

** During 1937-1938, confirmation was obtained from various sources of the existence of particles lighter than a proton but some 200 times heavier than the electron in the upper atmosphere initiated by cosmic ray radiation. Such particles were found to carry either a positive or a negative charge. Anderson proposed to call them mesotrons, from the Greek meso, meaning intermediate, since their mass lies between that of an electron and a proton. Later, the abbreviated name meson, suggested by the Indian physicist H.J. Bhabha in 1939, was adopted.

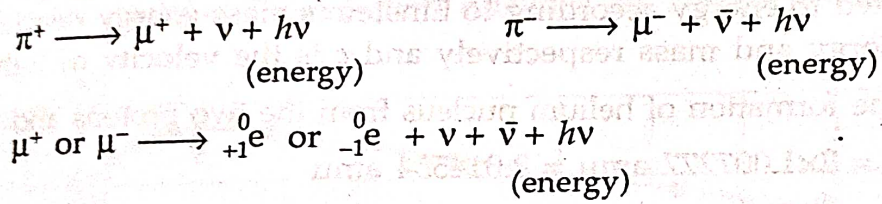
One of the characteristic of the particle now called meson is that they have a spin quantum number either zero or integral. The μ -meson (μ), however, has a half unit of spin and so it is not a meson in the current sense of the term. The π particles has a spin of zero and so is a meson, it is commonly referred to as π -meson or pion.

positive pi-meson (π^+), negative pi-meson (π^-) and neutral pi-meson (π^0). The π^0 have a mass 264 times that of the mass of the electron and the π^+ , π^- have masses 200 times greater than electron and possess single unit of positive and negative charges respectively. Yukawa proposed the following interconversions between proton and neutron in which charged mesons acting as an exchanging material to develop binding energy between the nucleons. Similarly π^0 acting as an exchanging material for similar nucleons i.e. between protons and neutrons.



Yukawa proposed in his meson theory that all the interconversions are in dynamic equilibrium to form the stable nucleus. Consequently a question arises why the heavy atomic number elements are unstable? It has been suggested that all the exchange forces operate within the small nuclear dimension, but for heavy nuclides the nuclear radius increases thereby the above nuclear exchange forces become weaker. In these cases the electrostatic repulsive forces predominate over the exchange forces and thus the nuclides become unstable.

Another type of meson was detected in cosmic rays which is known as μ -meson or muons and these have mass = $206 m_e$ where m_e = mass of electron and charge equal to either unit positive or negative charge. Outside the nucleus the π mesons are converted to μ -meson and they exchange energy in the following way :



Apart from π and μ mesons other type of mesons viz. k -meson or kaons, ρ -meson, η -meson, ω -meson and σ -meson have been discovered. The k -meson has mass = $966 m_e$. ω -meson involve in the repulsion between the nucleons. However these mesons play no principal role in nuclear forces like that of π -mesons. **All mesons are of short lived.** They are produced artificially in cyclotrons and characterised by scattering experiments.

Other particles characterised by the interaction of matter with high energy projectiles are— antielectron, antiproton, antineutron, hyperon etc.

9.12 MASS DEFECT AND NUCLEAR BINDING ENERGY

The next question in the problem of nuclear forces is how the packed combination of fundamental particles like neutron, proton, electron etc. strongly bind to form stable nucleus as also extranuclear part. Primarily a tremendous force of attraction operated between the neutron and the proton in the nucleus. The well established theory for the stability of the nucleus is that the isotopic mass (M) of an element is always less than the combined masses (M') of protons, neutrons, as also electrons. Thus the difference between the expected mass and the actual mass of an isotope is known as mass defect. Thus,

mass defect (ΔM) = Sum of the masses of protons (m_p), neutrons (m_n) and electrons (m_e) i.e. calculated mass (M') – actual mass of an isotope (M)

so that $\Delta M = M' - M$

Mass defect of an isotope A_ZX having mass number A and atomic number Z can be determined in the following way :

As $Z =$ number of protons (p) = number of electrons (e)

$$A = n + p \text{ and } n = A - Z$$

Therefore the isotope of the atom X contains Z protons, Z electrons and $(A-Z)$ neutrons. Then the calculated mass (M') for the Z number of protons, Z number of electrons and $(A-Z)$ number of neutrons will be,

$$M' = Zm_p + Zm_e + (A-Z)m_n$$

where m_p , m_e and m_n are the masses of proton, electron and neutron respectively. Now,

$$M' = Z(m_p + m_e) + (A-Z)m_n = Zm_H + (A-Z)m_n$$

because $m_p + m_e =$ mass of one hydrogen atom (m_H)

$$\text{Therefore } \Delta M = Zm_H + (A-Z)m_n - M \quad \dots(9.26)$$

Generally mass of the electron is very small so that mass defect can be represented as

$$\Delta M = (A-Z)m_n + Zm_p - M \quad \dots(9.27)$$

This mass (ΔM) is converted to energy according to Einstein's mass-energy relationship $E = mc^2$ where E and m are the energy and mass respectively and c is the velocity of light.

Now let us consider the formation of helium nucleus from the two protons and two neutrons :

$$\text{mass of two protons} = 2 \times 1.007277 \text{ amu} = 2.014554 \text{ amu}$$

$$\text{mass of two neutrons} = 2 \times 1.008665 \text{ amu} = 2.017330 \text{ amu}$$

$$\text{Total mass} = 4.031884 \text{ amu}$$

$$\text{Actual mass of } {}^4_2\text{He} = 4.0026 \text{ amu}$$

$$\text{Therefore } \Delta M = 4.031884 - 4.0026 = 0.029284 \text{ amu}$$

This amount of mass defect (0.029284 amu) is converted to energy to bind two protons and two neutrons to form the stable ${}^4_2\text{He}$. Therefore nuclear binding energy (NBE) is defined as the amount of energy released to bind neutrons and protons to form the nucleus and the nuclear binding energy per nucleon (\bar{B}) is defined as :

$$\bar{B} = \frac{\text{NBE}}{\text{Total number of nucleons}} \quad \dots(9.28)$$

Now we have to deduce the conversion factor for atomic mass unit (amu) to energy.

$$\begin{aligned} 1 \text{ amu or } 1 \text{ u or } 1 m_u &= \frac{1}{12} \times \frac{\text{mass of 1 mole of } {}^{12}\text{C atom}}{\text{Avogadro's number}} \\ &= \frac{1}{12} \times \frac{12}{6.023 \times 10^{23}} \text{ g} = 1.66 \times 10^{-24} \text{ g} = 1.66 \times 10^{-27} \text{ kg} \end{aligned}$$

According to $E = mc^2$, the energy equivalent to 1 amu is

$$E = mc^2 = 1.66 \times 10^{-27} \text{ kg} (2.998 \times 10^8 \text{ ms}^{-1})^2 = 14.92 \times 10^{-11} \text{ J}$$

$$\begin{aligned}
 &= \frac{14.92 \times 10^{-11} \text{ J}}{1.602 \times 10^{-19}} \text{ eV} \quad [\because 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}] \\
 &= 9.3133 \times 10^8 \text{ eV} = 931.33 \text{ MeV} \quad [\because 1 \text{ MeV} = 10^6 \text{ eV}] \\
 &\approx 931 \text{ MeV.}
 \end{aligned}$$

so that, $1 \text{ amu} \approx 931 \text{ MeV.}$

Therefore in case of ${}^4_2\text{He}$, $\bar{B} = \frac{0.029284 \times 931.33}{4} = 6.82 \text{ MeV.}$

The plot of \bar{B} against mass number of elements (Fig. 9.10) reveals the following points :

- (a) The **medium nuclides** having $A = 30-90$ registered highest \bar{B} . In this range the transition metals viz. iron, cobalt, nickel etc. having mass number 50-60 exist. These nuclides register highest stability and the highest stability zone is recorded by $\bar{B} = 8.5 \pm 0.2 \text{ MeV.}$
- (b) For **light elements** ($A < 30$) \bar{B} rises steeply. These nuclides e.g. ${}^4_2\text{He}$, ${}^{12}_6\text{C}$, ${}^{16}_8\text{O}$, ${}^{28}_{14}\text{Si}$ etc. are stable according to Oddo-Harkins rule (Sec.9.10). Exception ${}^8_4\text{Be}$ which splits into two α -particles i.e. ${}^8_4\text{Be} \rightarrow 2 {}^4_2\text{He}$. However the lighter nuclides having odd number of nucleons e.g. ${}^1_1\text{H}$, ${}^2_1\text{H}$ etc. undergo nuclear fusion* with liberation of energy to form stable ${}^4_2\text{He}$ nuclide. These lighter nuclides are susceptible to nuclear fusion.

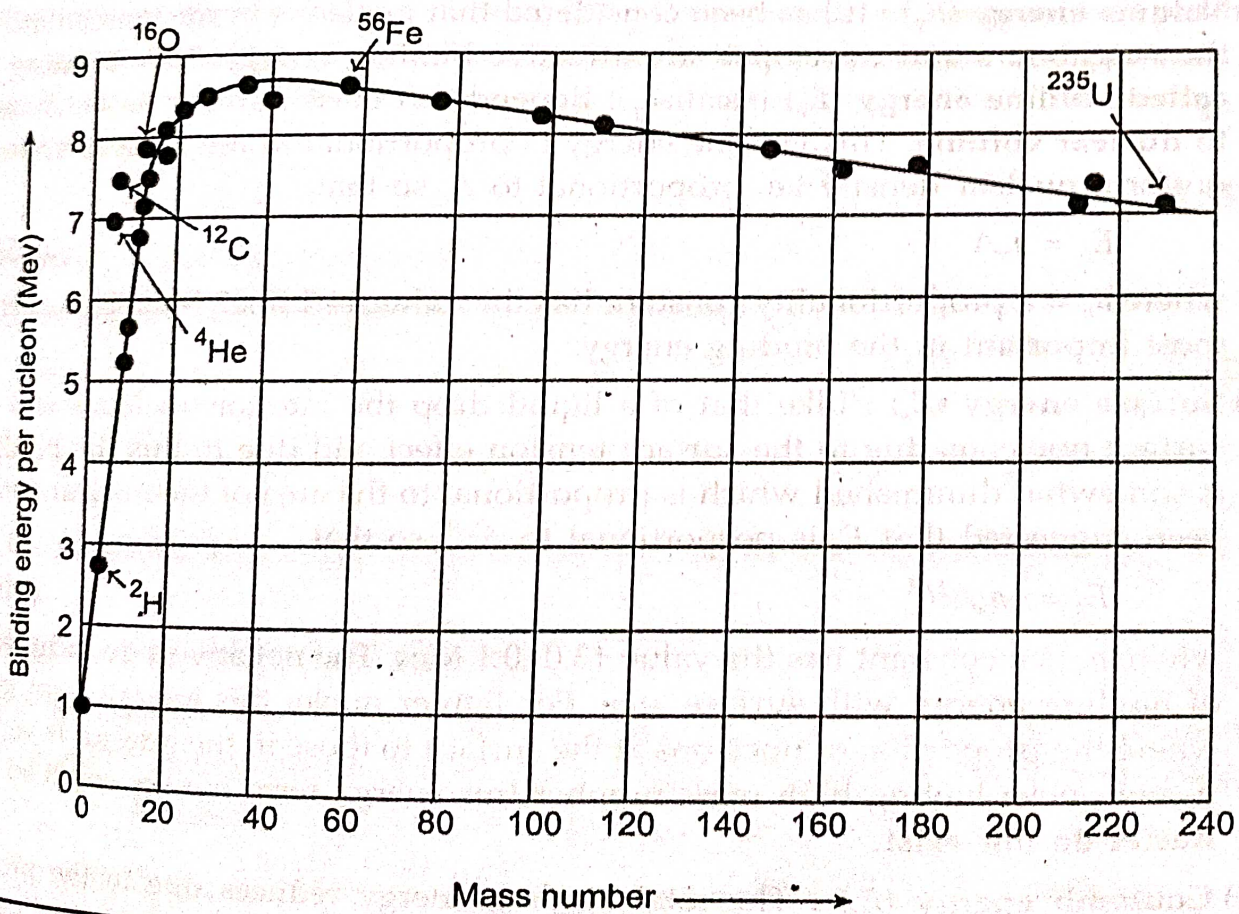


Figure 9.10 : Variation of binding energy per nucleon with mass number.

* will be discussed later.

(c) The heavy nuclides ($A > 90$) the \bar{B} gradually falls from the highest stability zone decreases from 8.7 to 7.5 Mev. After $A = 210$ the nuclides are very unstable and prone to disintegrate naturally (natural radioactivity) and these elements also undergo nuclear fission* which results in the release of energy. In this process heavy nuclides splitted into the smaller fragments thereby enter into the stability zone of binding energy. In fact ${}^{209}_{83}\text{Bi}$ is the heaviest non-radioactive stable nuclide. The stable end product of all natural radioactive nuclide is the isotope of lead which enters into the stability zone e.g. ${}^{235}_{92}\text{U}$ has $\bar{B} = 7.6$ Mev whereas ${}^{207}_{82}\text{Pb}$ has the $\bar{B} = 8.5 \pm 0.2$ Mev.

9.13.2 Particle Accelerators

As mentioned earlier projectiles should have required kinetic energy to carry out nuclear reactions. This is done by particle accelerators. First ingenious device was developed by Cockroft Walton (1931) to accelerate proton. The device was nothing but a voltage multiplier. Later most widely used particle accelerator was developed by E.O. Lawrence in 1932 in the University of California which is known as cyclotron. E.O. Lawrence received Nobel Prize for his ingenious device. Different types of modern particle accelerators such as betatron, proton synchrotron, electron synchrotron, synchrocyclotron etc. are now available.

9.13.2.1 The Cyclotron

In this machine charged particles such as protons, deuterons, α -particles etc. are accelerated by simultaneous application of electric and magnetic field. It consists of two semicircular hollow boxes (D_1 and D_2) known as 'dees' (Fig. 9.12). The two 'dees' are separated by few centimeter apart and enclosed in a large chamber. These 'dees' are connected to a source of high voltage alternating current and are placed over a powerful electromagnet. At the centre of the two 'dees' there is an electrically heated filament (S) which emits thermoionic emission.

Initially the apparatus is evacuated and appropriate gases such as hydrogen, helium or deuterium etc. are introduced at low pressure. Thermoionic emission at S will produce electron beam and this on bombardment to the appropriate gas produces proton, α -particle etc. as the case may be. The positive ions then accelerated into the 'dees' D_1 and D_2 .

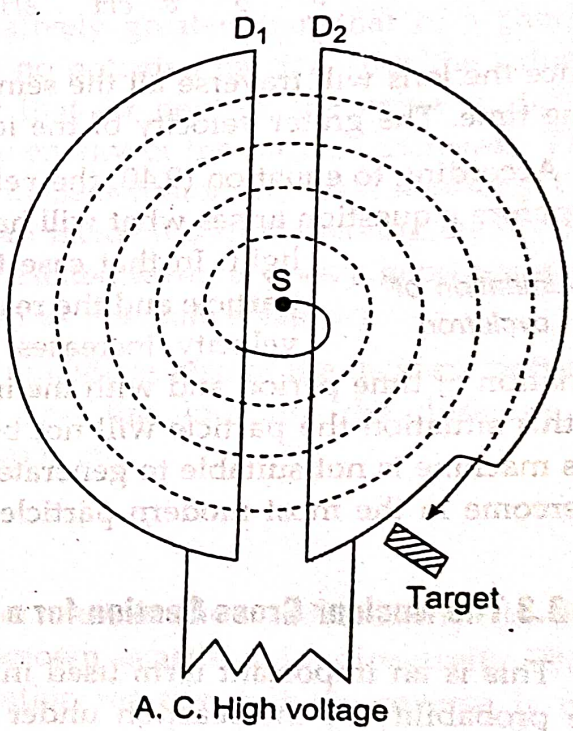


Figure 9.12 : Schematic diagram of a cyclotron.

Suppose at first D_2 is negative, then positive ions at S will move to D_2 . But the magnetic field which acts perpendicular to the electric field and thus the positive ions move in the circular path of D_2 . As the ions come at S , the oscillating high voltage changes its sign thereby D_1 becomes quickly negative and D_2 becomes positive so that positive ions will move from D_2 to D_1 . Thus the process is gradually repeated, the positive ions will acquire high kinetic energy (~ 40 MeV or α -particles) and ultimately the ions emerged from the periphery of one of the 'dees' and made to strike on a target. The basic condition to do this is that to use high frequency alternating potential.

Let the magnetic field strength H causes the movement of positive ions in semicircular 'dees' in a circular path of radius r with constant velocity v , so that,

$$Hev = \frac{mv^2}{r} \text{ i.e. } \frac{e}{m} = \frac{v}{Hr} \text{ so that, } r = \frac{vm}{eH} \quad \dots(9.40)$$

where m = mass of the positive ions and e is the electronic charge. This equation (9.40) indicates that $r \propto v$ when other factors are constant. This means that kinetic energy of the positive ions increases with increasing radius of the machine. Brookhaven National Laboratory and University of California has the largest and most sophisticated particle accelerators.

One of the important feature of cyclotron is that the time period (t) to traverse the semicircular 'dees' is independent of both r and v for all ions with same mass and charge. Since the semicircular trajectories are half circle, so that

$$\begin{aligned} t &= \frac{\pi}{\omega} \text{ where } \omega \text{ is the angular velocity and } v = \omega r \\ &= \frac{\pi}{\frac{v}{r}} = \frac{\pi r}{v} = \frac{\pi}{v} \cdot \frac{vm}{eH} = \frac{\pi m}{eH} \end{aligned} \quad \dots(9.41)$$

Hence the ions will traverse all the semicircular 'dees', whatever be their radius, in exactly the same time. The greater velocity of the ions compensate for the longer path to be traversed.

According to equation (9.40) the velocity i.e. kinetic energy increases with the increase of r . Therefore a question arises what will happen when the velocity is comparable to the velocity of light. In that case the relativistic correction (Sec. 1.7.1) of the mass of the particle and the rest mass (m_0) will not be equal to moving mass (m). As the velocity increases, the mass of the particle increases, there will be the variation of time period and with the increase of linear velocity, the angular velocity decreases. In this situation the particle will not be able to move faster from D_1 to D_2 or vice-versa. Thus this machine is not suitable to generate very high kinetic energy of projectiles. This difficulty is overcome in the most modern particle accelerator known as synchrocyclotron.

9.13.3 The Nuclear Cross Section for a Nuclear Reaction

This is an important term used in nuclear reactions. The nuclear cross section (σ) denotes the probability of the reaction under consideration. σ also denotes the yield of the reaction which is defined as the ratio of the number of emitted particles to the number of incident particles in a reaction.

Let I = total number of incident particles which penetrate completely a unit area of a thin target in a given time.

t = thickness of the target in cm.

N = number of particles per sq. cm of the target which will interact with a beam of incident particles perpendicularly to the target.

m = the number of the target nuclei reacting effectively in a given time for a type of reaction considered which is also the number of collisions per cm^2 per second.

The average volume per nucleus in the target = $\frac{1 \times l}{N} \text{ cm}^3$.

Since each projectile particle will interact with target nuclei having cylindrical volume of cross section σ and length equal to its depth of penetration i.e. l cm, so that effective volume of the target interact with I number of incident particles in traversing the target = $I\sigma l \text{ cm}^3$.

Thus the number of collisions (m) = $\frac{I\sigma l \text{ cm}^3}{\frac{l}{N} \text{ cm}^3} = IN\sigma$

so that $\sigma = \frac{m}{IN} \text{ cm}^2$

...(9.42)

The range of σ for various nuclear reactions is in between 10^{-20} to 10^{-32} cm^2 . σ generally expressed in the units of barn (b). $1\text{b} = 10^{-24} \text{ cm}^2 = 10^{-28} \text{ m}^2 = 10^2 \text{ fm}^2$ where $\text{fm} = \text{fermi}$.

A high value of σ indicates greater efficiency of a nuclear reaction. The value of σ depends not only on the type of reaction but also on the energy of the incident particles. It has been established that in case of charged particles the nuclear cross section increases with increasing energy of the projectile. For neutron the σ is comparatively greater than that of a charged particle. This is due to the fact that for neutron there is no potential barrier. Thus the value of σ lies in the range 10^{-29} to 10^{-27} cm^2 for proton while that for neutron the value is 10^{-26} to 10^{-24} cm^2 . In case of neutron σ generally decreases as the energy of the particle increases. Thus slow neutrons are very good projectile for carrying out nuclear fission than fast neutrons. One important aspect of the energy of the projectile is that an incident particle having energy just sufficient to excite the compound nucleus to any of the excited level, is a very good projectile for carrying out effective nuclear reaction. This is known as resonance capture.

The σ also depends on the atomic numbers of target nuclide as also of the incident particle. The value decreases as the atomic number increases and this indicates the difficulty in transmutation of heavy nuclides with α -particles, deuterons etc.

9.13.4 Artificial Radioactivity

In some artificial transmutation reactions if the product nuclides show radioactivity like that of natural radioactive elements, then the phenomenon is known as artificial radioactivity. Thus all artificial radioactivity are termed artificial transmutation reaction but the reverse is not always true.

In 1934 Irene Curie (daughter of Madam Curie) and her husband Frederick Joliot was studying artificial transmutation reactions with some light nuclides. With much curiosity they observed that when boron, aluminium, magnesium etc. were bombarded with α -particles two nuclear paths were observed. Thus when aluminium target was bombarded with α -particle in one path protons were the ejectile. In the second path an isotope of phosphorous i.e. $(^{30}_{15}\text{P})$ were produced along with neutron was ejectile. It was noticed that as soon as the source of α -particles was withdrawn, the proton and neutron emission were ceased. But the emission of positron continued for a while although the rate of emission decreases with time which follow first order kinetics in accordance with that of natural radioactive decay process. From these